PreCalc: 3.2 Polynomial Functions and Models

Name________________________________ Date:________________ period:______

Summary #1 (p.178):

Graph of a Polynomial Function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$, $a_n \neq 0$

- Degree of the polynomial *f*: n
- Maximum number of turning points: n – 1
- At a zero of even multiplicity: The graph of *f* touches the x-axis.
- At a zero of odd multiplicity: The graph of *f* crosses the x-axis.
- Between zeros, the graph of *f* is either above or below the x-axis.
- End Behavior: For large |x|, the graph of *f* behaves like the graph of $y = a_n x^n$
  Ex. For $f(x) = -3x^5 - 4x^3 - 7x^2 + 2$, find the degree of the polynomial. Determine the end behavior; that is, find the power function that the graph of *f* resembles for large values of |x|. Degree = 5, End Behavior = $-3x^5$ for large values of |x|.

Summary #2 (p.181):

Steps for graphing a polynomial by hand

To analyze the graph of a polynomial function y = f(x), follow these steps:

1. End behavior: find the power function that the graph of *f* resembles for large values of x.
2. A. Find the x-intercepts, if any, by solving the equation f(x) = 0
   B. Find the y-intercept by letting x = 0 and finding the value of f(0).
3. Determine whether the graph of *f* crosses or touches the x-axis at each x-intercept
4. Use a graphing utility to graph f. Determine the number of turning points on the graph of f. Approximate, using the graphing utility, any turning points rounded to two decimal places.
5. Use the information obtained in steps 1-4 to draw a complete graph of *f* by hand.

**** See examples 7 and 8 in the text on p.179-180 for how to utilize the 2 summaries above in a given polynomial problem.**

EXAMPLE 8 **Using a Graphing Utility to Analyze the Graph of a Polynomial Function**

For the polynomial $f(x) = x^3 + 2.48x^2 - 4.3155x + 2.484406$:

(a) Find the degree of the polynomial. Determine the end behavior: that is, find the power function that the graph of f resembles for large values of $|x|$.

(b) Graph f using a graphing utility.

(c) Find the x- and y-intercepts of the graph.

(d) Use a TABLE to find points on the graph around each x-intercept.

(e) Determine the local maxima and local minima, if any exist, rounded to two decimal places. That is, locate any turning points.

(f) Use the information obtained in parts (a)–(e) to draw a complete graph of f by hand. Be sure to label the intercepts, turning points, and the points obtained in part (d).

(g) Find the domain of f. Use the graph to find the range of f.

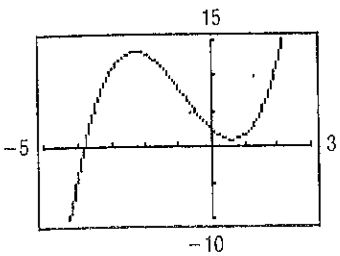
(h) Use the graph to determine where f is increasing and where f is decreasing.

Solution

(a) The degree of the polynomial is 3. End behavior: the graph of f resembles that of the power function $y = x^3$ for large values of $|x|$.

(b) See Figure 34 for the graph of f.

Figure 34



(c) The y-intercept is $f(0) = 2.484406$. In Example 7 we could easily factor $f(x)$ to find the x-intercepts. However, it is not readily apparent how $f(x)$ factors in this example. Therefore, we use a graphing utility's ZERO (or ROOT) feature and find the lone x-intercept to be -3.79, rounded to two decimal places.

(d) Table 7 shows values of x around the x-intercept. The points $(-4, -4.57)$ and $(-2, 13.04)$ are on the graph.

Table 7

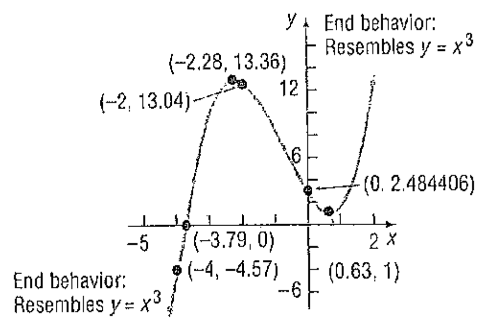
X	Y1	
-4	-4.574	
-2	13.035	

Y1=X^3+2.48X²-4...

(e) From the graph we see that it has two turning points: one between -3 and -2 the other between 0 and 1. Rounded to two decimal places, the local maximum is 13.36 and occurs at $x = -2.28$; the local minimum is 1 and occurs at $x = 0.63$. The turning points are $(-2.28, 13.36)$ and $(0.63, 1)$.

(f) Figure 35 shows a graph of f drawn by hand using the information obtained in parts (a) to (e).

Figure 35



(g) The domain and the range of f are the set of all real numbers.

(h) Based on the graph, f is decreasing on the interval $(-2.28, 0.63)$ and is increasing on the intervals $(-\infty, -2.28)$ and $(0.63, \infty)$.

NOW WORK PROBLEM 81.

Precalc Classwork Sec. 3.2

Name ____________________

Mod _____

1) For the polynomial: $f(x) = -x^2(x^2 - 4)(x - 5)$

(a) Find the degree of the polynomial. Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

(b) Find the x- and y-intercepts of the graph of f.

(c) Determine whether the graph crosses or touches the x-axis at each x-intercept.

(d) Use a graphing utility to graph f.

(e) Determine the number of turning points on the graph of f. Approximate the turning points, if any exist, rounded to two decimal places.

(f) Use the information obtained in parts (a) to (e) to draw a complete graph of f by hand.

(g) Find the domain of f. Use the graph to find the range of f.

(h) Use the graph to determine where f is increasing and where f is decreasing.

2) For the polynomial $f(x) = -1.2x^4 + 0.5x^2 - \sqrt{3}x + 2$

(a) Find the degree of the polynomial. Determine the end behavior: that is, find the power function that the graph of f resembles for large values of $|x|$.

(b) Graph f using a graphing utility.

(c) Find the x- and y-intercepts of the graph.

(d) Use a TABLE to find points on the graph around each x-intercept.

(e) Determine the local maxima and local minima, if any exist, rounded to two decimal places. That is, locate any turning points.

(f) Use the information obtained in parts (a)–(e) to draw a complete graph of f by hand. Be sure to label the intercepts, turning points, and the points obtained in part (d).

(g) Find the domain of f. Use the graph to find the range of f.

(h) Use the graph to determine where f is increasing and where f is decreasing.

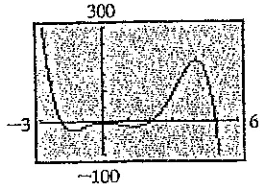
PRECALC W.S. 3.2 (ANSWERS)

1) (a) Degree 5; $y = -x^5$

(b) x-intercepts: $-2, 0, 2, 5$; y-intercept: 0

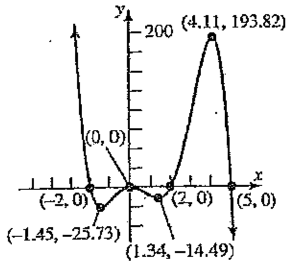
(c) 0: Touches; $-2, 2, 5$: Crosses

(d)



(e) Local minima at $(-1.45, -25.73)$, $(1.34, -14.49)$; local maxima at $(0, 0)$, $(4.11, 193.82)$

(f)

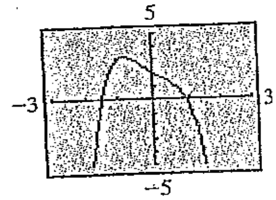


(g) Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

(h) Increasing on $(-1.45, 0)$ and $(1.34, 4.11)$
Decreasing on $(-\infty, -1.45)$, $(0, 1.34)$, and $(4.11, \infty)$

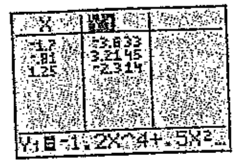
2) (a) Degree 4; $y = -1.2x^4$

(b)



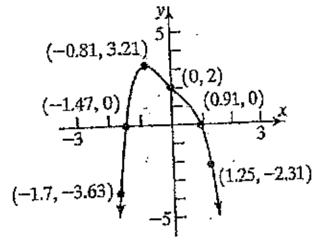
(c) x-intercepts: $-1.47, 0.91$
y-intercept: 2

(d)



(e) Local maximum at $(-0.81, 3.21)$

(f)



(g) Domain: $(-\infty, \infty)$
Range: $(-\infty, 3.21]$

(h) Increasing on $(-\infty, -0.81)$
Decreasing on $(-0.81, \infty)$